Finite-difference time-domain method

for electromagnetics

Maxwell's equations (Lorentz Heaviside units)

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \qquad \mathbf{D} = \epsilon \mathbf{E}$$

$$\nabla \times \mathbf{H} = \frac{1}{c}\left(\frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}\right) \qquad \mathbf{B} = \mu \mathbf{H}$$

Updating the electric and magnetic fields

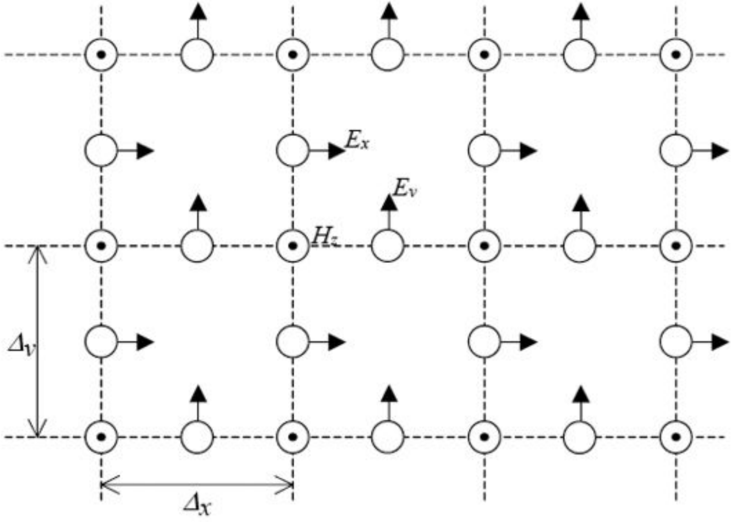
$$\partial_t \mathbf{B} = -c\nabla \times \mathbf{E}$$

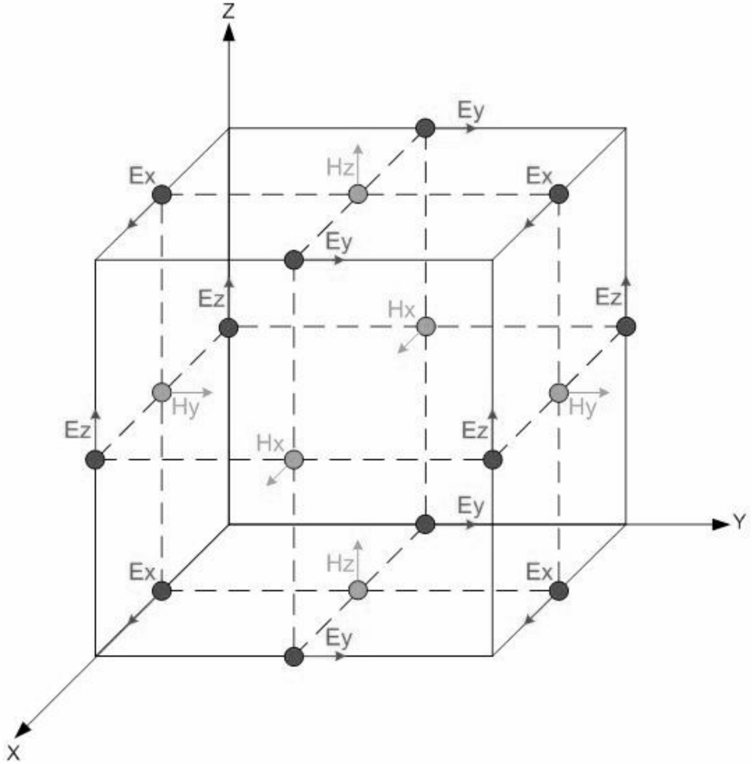
$$\partial_t \mathbf{B}_x = c(\partial_z \mathbf{E}_y - \partial_y \mathbf{E}_z)$$
$$\partial_t \mathbf{B}_y = c(\partial_x \mathbf{E}_z - \partial_z \mathbf{E}_x)$$
$$\partial_t \mathbf{B}_z = c(\partial_y \mathbf{E}_x - \partial_x \mathbf{E}_y)$$

$$\partial_t \mathbf{D} = c\nabla \times \mathbf{H} - \mathbf{J}$$

$$\partial_t \mathbf{D}_x = c(\partial_y \mathbf{H}_z - \partial_z \mathbf{H}_y) - \mathbf{J}_x$$
$$\partial_t \mathbf{D}_y = c(\partial_z \mathbf{H}_x - \partial_x \mathbf{H}_z) - \mathbf{J}_y$$
$$\partial_t \mathbf{D}_z = c(\partial_x \mathbf{H}_y - \partial_y \mathbf{H}_x) - \mathbf{J}_z$$

Finite difference method - staggered grid (Yee cell)





Absorbing boundary conditions

Absorbing Boundary Conditions for the Numerical Simulation of Waves

Bjorn Engquist, Andrew Majda

Absorbing Boundary Conditions for the Finite-Difference Approximation of the Time-Domain Electromagnetic Field Equations

Gerrit Mur

$$(\partial_x - c^{-1}\partial_t)W|_{x=0} = 0$$

Anisotropic materials

Numerical solution of initial boundary value problems involving maxwell's equations in isotropic media

Kane Yee

ϵ and μ are tensors

$$\mathbf{D}_i = \sum_j \epsilon_i^j \mathbf{E}_j \qquad \mathbf{B}_i = \sum_j \mu_i^j \mathbf{H}_j$$

Nonlinear optics

Dielectric polarization **P** responds nonlinearly to **E**

Second and third harmonic generation

Optical Kerr effect (self-focusing and optical solitons)

$$\mathbf{D} = \mathbf{E} + \mathbf{P} \qquad \mathbf{B} = \mathbf{H} + \mathbf{M}$$