Quantum machine learning

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Overview

- Research question
  - Background and problem of interest
  - Current challenges in addressing the problem
  - Hypothesis and rationale for formulating hypothesis
  - Specific aims
- Background
- Professor Alfred Aho
  - Current research
  - Interested projects and contributions

Research question background

What is a bit?

- Unit of classical information
- Can be **either 0 or 1**

H	01001000	**W**	01010111
E	01000101	**O**	01001111
L	01001100	**R**	01010010
L	01001100	**L**	01001100
O	01001111	**D**	01000100

Research question background

What is a gate?

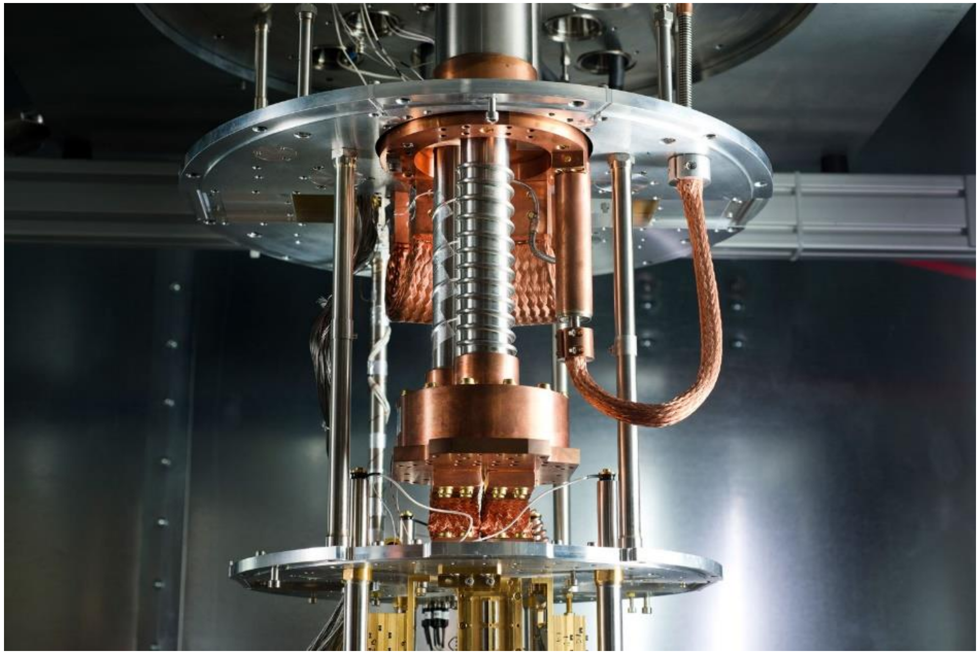
- Operation on bits

AND 0 0 = 0	**OR** 0 0 = 0	**XOR** 0 0 = 0	**NOT** 0 = 1
AND 0 1 = 0	**OR** 0 1 = 1	**XOR** 0 1 = 1	**NOT** 1 = 0
AND 1 0 = 0	**OR** 1 0 = 1	**XOR** 1 0 = 1	
AND 1 1 = 1	**OR** 1 1 = 1	**XOR** 1 1 = 0	

Research question background

What is a quantum computer?

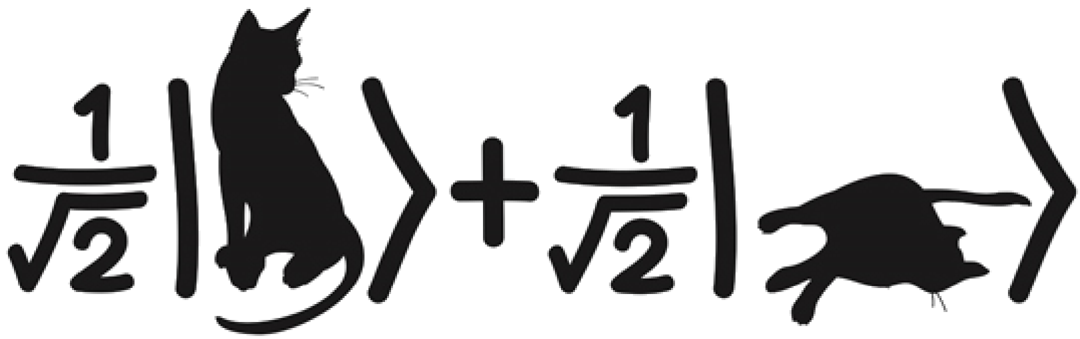
- Quantum states and gates
- **Superposition and entanglement**



Research question background

What is a qubit?

- Unit of quantum information
- Can be in a **superposition of 0 and 1**



- |0⟩ and |1⟩ are called **basis states**
- Can be implemented by a **two-state quantum system**, e.g. charge, spin state, current direction, energy state

Research question background

State is **linear combination** of $|0\rangle$ and $|1\rangle$

$$|x\rangle = a|0\rangle + b|1\rangle$$

where a and b are **probability amplitudes**

Probability of measuring 0 is $|a|^2$

Probability of measuring 1 is $|b|^2$

Probabilities sum to unity: $|a|^2 + |b|^2 = 1$

Research question background

Combined state is **tensor product** of states

$$|xy\rangle = |x\rangle \otimes |y\rangle$$

and thus linear combination of combined basis states

$$|xy\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

where $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$

Research question background

Let $|x\rangle = a|0\rangle + b|1\rangle$

Let $|y\rangle = c|0\rangle + d|1\rangle$

The combined state is

$$
\begin{aligned}
|xy\rangle &= |x\rangle \otimes |y\rangle \\
&= (a|0\rangle + b|1\rangle) \otimes (c|0\rangle + d|1\rangle) \\
&= a|0\rangle \otimes c|0\rangle + a|0\rangle \otimes d|1\rangle + b|1\rangle \otimes c|0\rangle + b|1\rangle \otimes d|1\rangle \\
&= ac|0\rangle \otimes |0\rangle + ad|0\rangle \otimes |1\rangle + bc|1\rangle \otimes |0\rangle + bd|1\rangle \otimes |1\rangle \\
&= ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle
\end{aligned}
$$

Research question background

What is a quantum gate?

- **Linear operation** on qubits
- **Unitary operation**, preserves **inner product**

$$\langle Ux|Uy\rangle = \langle x|y\rangle$$

$$U^\dagger U = UU^\dagger = I$$

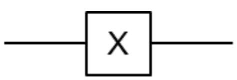
where $U^\dagger$ is **conjugate transpose** of U

- Hence U is **reversible** ($U^{-1} = U^\dagger$)

Research question background

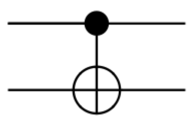
What is a **quantum circuit**?

- Computation as a sequence of quantum gates
- Operate on a **quantum register** (collection of qubits)



NOT $|0\rangle = |1\rangle$

NOT $|1\rangle = |0\rangle$

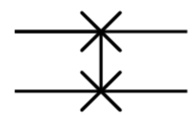


CNOT $|00\rangle = |00\rangle$

CNOT $|01\rangle = |01\rangle$

CNOT $|10\rangle = |11\rangle$

CNOT $|11\rangle = |10\rangle$



SWAP $|00\rangle = |00\rangle$

SWAP $|01\rangle = |01\rangle$

SWAP $|10\rangle = |10\rangle$

SWAP $|11\rangle = |11\rangle$

Research question background

What is the Hadamard gate?

- Creates an equal superposition of basis states

$$\mathbf{H}|1\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

$$(\mathbf{H}|0\rangle)^{\otimes n} = \frac{1}{\sqrt{2^n}} \sum_{k=0}^{2^n-1} |k\rangle$$

where $|k\rangle$ is the binary state representation of k

Research question background

What is a phase-shift gate?

- Changes phases, not probabilities, of basis states

$\mathbf{R_\theta}\ |0\rangle = |0\rangle$

$\mathbf{R_\theta}\ |1\rangle = e^{i\theta}\ |1\rangle$

The Hadamard and phase-shift gates can be combined to generate any pure qubit state, up to a global phase

$\mathbf{R_{\pi/2+\varphi}\ H\ R_{2\theta}\ H}\ |0\rangle = \cos\theta\ |0\rangle + e^{i\varphi} \sin\theta\ |1\rangle$

Research question background

Quantum Fourier transform: computes discrete Fourier transform in $O(n^2)$ gates, classical version requires $O(n2^n)$

Maps $(x_0, x_1, \ldots, x_{n-1})$ to $(y_0, y_1, \ldots, y_{n-1})$

$$y_j = \frac{1}{\sqrt{n}} \sum_{k=0}^{n-1} e^{\frac{2\pi i}{n} jk} x_k$$

$$|j\rangle \rightarrow \frac{1}{\sqrt{n}} \sum_{k=0}^{n-1} e^{\frac{2\pi i}{n} jk} |k\rangle$$

Research question background

Grover's algorithm: searches unsorted database with n entries in $O(n^{1/2})$ time, linear search requires $O(n)$

Shor's algorithm: finds prime factors of n in $O((\log n)^3)$ time, GNFS requires $O(\exp(c(\log n)^{1/3}(\log \log n)^{2/3}))$

Breaks public key cryptography schemes, such as RSA

Search for quantum and post-quantum cryptography

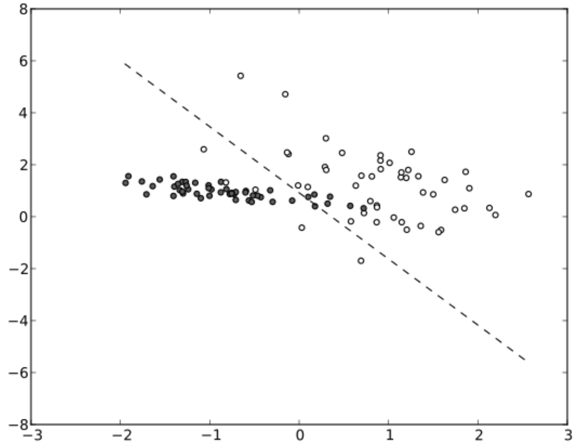
Research question and hypothesis

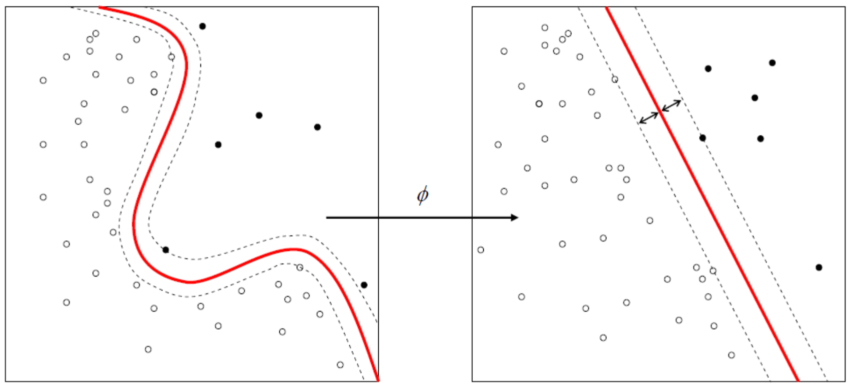
- How can quantum computing be used to improve the efficiency of machine learning techniques?
  - Efficient calculation of distances between data points (for use in SVMs and k-nearest neighbor methods)
  - Quantum models or formulations of ANNs, Bayesian networks, and hidden Markov models for problem-solving
  - Formulating decision strategies in terms of quantum physics
  - Implementing optimization problems (usually solved by iterative gradient descent) on a quantum computer
- **How can the efficiency of support vector machines be improved through quantum computing?**

Specific aims

Quantum computing for support vector machines:

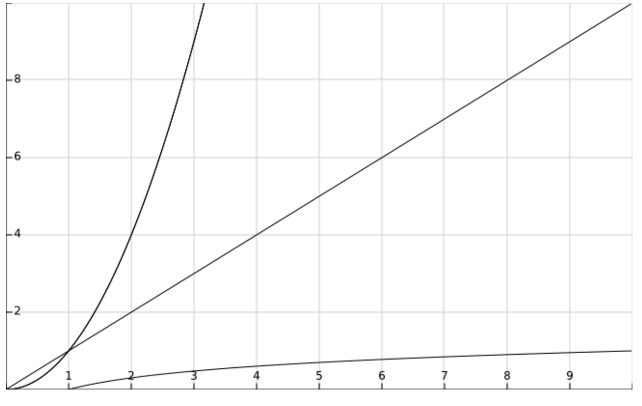
- Used for linear discrimination, pattern classification
- Finding hyperplane that best discriminates between two class regions containing data points
- Non-linear problems can be mapped to linear ones in a higher dimensional space (**kernel method**)

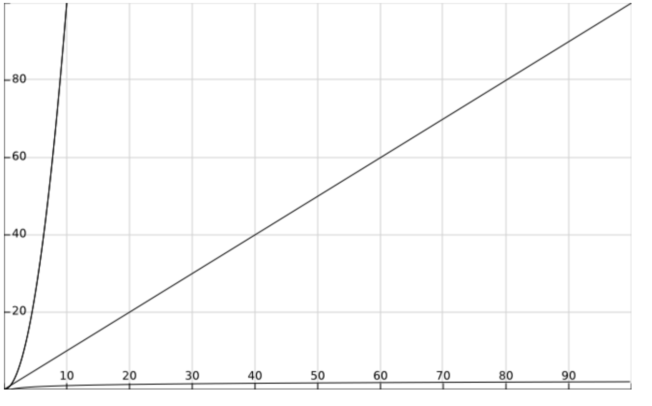




Specific aims

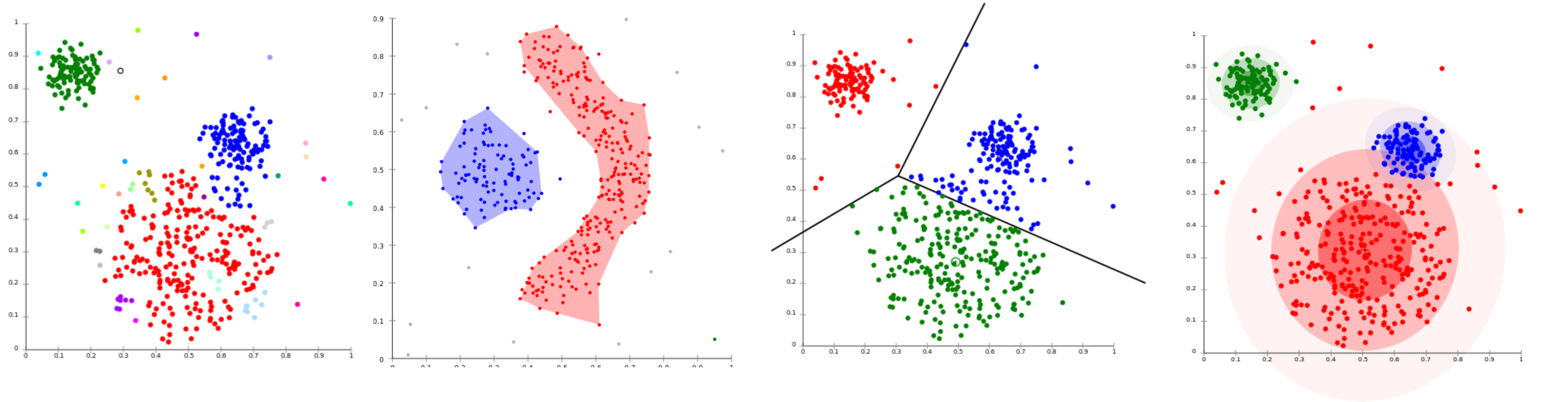
- Manipulating and classifying large number of vectors in high-dimensional spaces (**feature spaces**)
- Quantum computers excel at manipulating vectors in large, high-dimensional tensor product spaces
- Performance in number of vectors and dimensions:
  - Classical algorithms: **polynomial**
  - Quantum algorithms: **logarithmic**





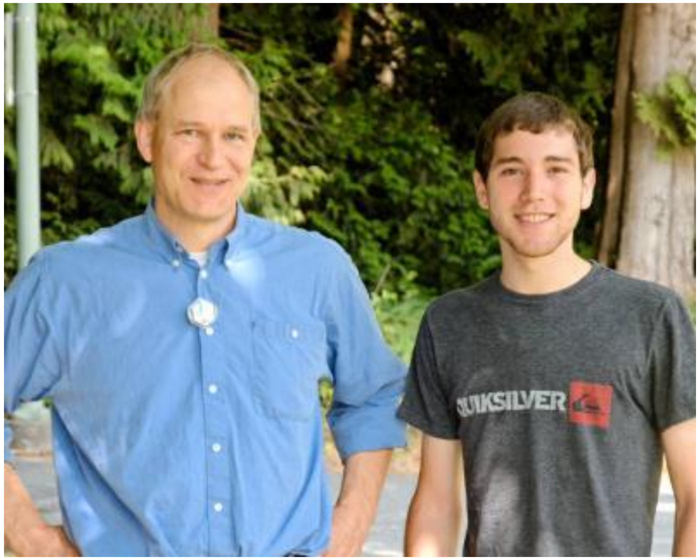
Specific aims

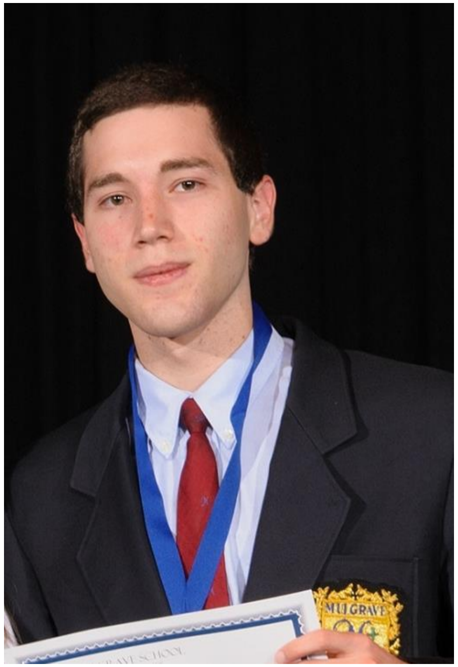
- Support vector machines rely on a kernel containing the inner product of feature vectors as entries
- Calculating kernels is computationally expensive
- **Inner product evaluation could be done faster on a quantum computer using quantum states**
- Optimization of inner product evaluation for SVMs and distance measures for **cluster analysis**



Background

- Education
  - Mulgrave School Class of 2014: IB Higher Level Physics, Chemistry, and Mathematics
  - Columbia University Class of 2018, Fu Foundation School of Engineering and Applied Science
    - Applied Physics and Computer Engineering





Background

- Honors
  - TRIUMF High School Fellowship recipient
  - Michael Smith National Science Challenge Top 3%
  - Certificate of Distinction in Pascal, Cayley, Fermat, Euclid, Canadian Senior Mathematics Contests
  - Canadian Open Mathematics Challenge Distinction
  - British Columbia Passport to Education Award
  - Gold medal in Venezuelan Mathematical Olympiad

Background

- Work or Research Experiences
  - High school fellowship at TRIUMF, Canada’s national laboratory for particle and nuclear physics
    - Development and operation of laser ion sources
    - Delivery of radioactive ion beams for use in experiments
    - Use of resonance ionization laser ion source (RILIS) to separate isobaric radioactive ion beams
    - Resonant transitions driven by a pulsed Ti-Sapphire laser pumped by frequency-doubled Nd:YAG lasers
    - ABCD ray transfer matrix analysis of laser system
    - Surface ionization properties (Saha-Langmuir equation)

Background

- Work or Research Experiences
  - Appazur solutions cross-platform mobile development
  - Orthogonal transformations in R^n, rotations through Euler angles, matrices, axis-angles, quaternions
  - Numerical methods for solving differential equations: Euler, symplectic Verlet, Midpoint, Runge-Kutta

Background

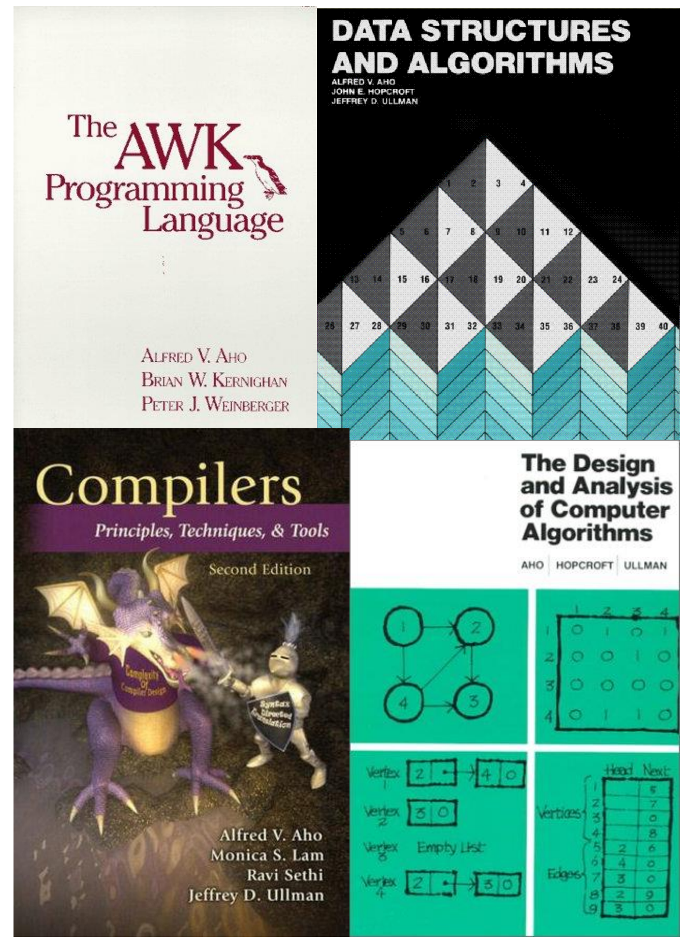
- Other independent research
  - Physics engine development for classical mechanics simulations, collision detection and resolution
  - Computational fluid dynamics using Eulerian (grid-based) and Lagrangian (particle-based) methods like Smoothed Particle Hydrodynamics (SPH)
  - Computational electromagnetics simulation based on finite-difference time-domain (FDTD) method
  - Machine learning through support vector machines (SVMs) and artificial neural networks (ANNs)
  - Programming language lexing, parsing, compiling

Background

- Research interests
  - Quantum computing and information science
  - Neuromorphic computing systems
  - Artificial intelligence and machine learning
- Future career goals
  - Graduate research in quantum computing
  - Application of research to optimization problems in physics, mathematics, and computer science
  - Application of research to cryptographic schemes

Professor Alfred Aho

- Research projects:
  - Columbia Language and Compilers Research Group
  - Software architectures for quantum computing design tools
  - Compiling quantum circuits (using Palindrome Transform)
  - Threshold estimates for fault-tolerant quantum computing



Interest

- Research projects of interest
  - General purpose quantum compiler: decomposition of unitary operations into elementary operations (e.g. single qubit rotations, CNOT gates) through CS Decomposition (example: Qubiter)
  - Implementation of imperative and functional languages
  - Efficient methods for incorporating fault tolerance and quantum error correction into programs (decoherence, quantum noise)
  - Efficient algorithms for optimizing and verifying programs
  - Synthesis and simulation of quantum circuits
  - Topological quantum computing (robust against noise)
  - Adiabatic quantum computing (Hamiltonian slowly evolved to one whose ground state encodes the solution)

Interest

- Potential contributions based on prior research experience or interests
  - Experience programming in C/C++, Java, Python, etc.
  - Experience simulating physical phenomena using computational methods (e.g. numerical integration, finite difference time-domain method, smoothed particle hydrodynamics)
  - Experience programming algorithms for machine learning, data mining, pattern recognition, classification
- Time commitment: 10 hours per week + summer months